Modeling, Control and Games through Partial Differential Equations

02–04.09.2025

Room A5, Polo Scientifico degli Adelardi
Via degli Adelardi 33, Ferrara

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Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
Università degli Studi di Ferrara

Scientific Committee:	Rinaldo M. Colombo Andrea Corli Mauro Garavello Luisa Malaguti
Organizing Committee:	Andrea Corli Massimiliano D. Rosini

Program

(35 minutes talk + 10 minutes questions)

02.09	Tuesday
14:00-14:30	Opening
14:30-15:15	R.M. Colombo
15:15-16:00	R. Rossi
16:00-16:30	Coffee
16:30-17:15	F. Bagagiolo
17:15-18:00	L. Caravenna

03.09	Wednesday
09:00-09:45	M. Garavello
09:45-10:30	ϟ Posters ϟ
10:30-11:00	Coffee
11:00-11:45	C. Rebelo
11:45-12:30	G. Guerra
14:15-15:00	A. Keimer
15:00-15:45	E. Rossi
15:45-16:15	Coffee
16:15-17:00	M.D. Rosini
17:00-17:45	E. Marchini
20:00	Social Dinner

04.09	Thursday
09:00-09:45	F. Ancona
09:45-10:30	C. Donadello
10:30-11:00	Coffee
11:00-11:45	L. Malaguti
11:45-12:30	H. Holden
12:30	Closing

⚡ Lightning Presentations ⚡ **(3 minutes)**

Diego Berti
On Hamilton-Jacobi Equations for the N-Body Problem

Gianmarco Cipollone
Decay of Periodic Solutions to an Euler-Alignment

Giovanni Giliberti
Non-local Evolution Equations in L^1 Spaces

Stefan Moreti
Study of some Conservation Laws with Hysteresis

Claudia Nocita
Nonlocal and Mixed Nonlocal-Local Multicomponent Traffic Flow Models

Andrea Salvadori
Non Local Balance Laws in Cryptography

Fatemeh G. Zefreh
Kermack-McKendrick Type Models for Epidemiology

Abstracts

On the optimization of traffic flow at a junction

Fabio Ancona – Università di Padova

We first introduce a general framework to analyze control problems for strictly concave conservation law models on a network. We regard as controls the distributional parameters at the junction and the inflow controls at the node. Within this framework we discuss the existence of optimal controls for various functionals depending on the value of the solution at the node, and along the incoming or outgoing edges.

Next, we focus on a min-max problem on a 1-1 network, with inflow controls acting at the junction. We investigate the minimization problem for a functional measuring the total variation of the flow of the solutions at the node, among those solutions that maximize the time integral of the flux. In the case the initial datum is monotone, we show that the flux of the entropy weak solution at the node provides an optimal inflow control for this min-max problem. We also discuss two prototype examples showing that, in the case where the initial datum is not monotone, the flux of the entropy weak solution is no more optimal.

Joint work with: Annalisa Cesaroni (University of Padova), Giuseppe M. Coclite (University of Bari) and Mauro Garavello (University of Milano Bicocca).

Program

Viscosity solutions on Hilbert spaces for centralized constrained optimal control and differential games of evolutionary masses

Fabio Bagagiolo – Università di Trento

In the articles [1] and [2] a differential game between a single player and a mass as well as between two masses are studied. A corresponding Hamilton-Jacobi-Isaacs equation is derived on an invariant set of a Hilbert space and studied in the framework of the viscosity solutions theory. More recently, similar techniques are applied in [3] to the case of the optimal control for a single mass under a constraint on its support. In all cases, the evolution of the masses is given by a first order partial differential equation in $\mathbb{R}^n$ (the continuity equation) which is controlled in a centralized manner by choosing the velocity field. A suitable spatial regularity of the admissible controls allows to restrict to the case where the mass has a density, given by a function on $\mathbb{R}^n$, and hence to cast the problem into a Hilbert setting. This is a joint research program with Rossana Capuani, Luciano Marzufero and Ivan Romanò.

References

[1] F. Bagagiolo, R. Capuani and L. Marzufero. A single player and a mass of agents: a pursuit evasion-like game. ESAIM COCV Control Optimisation and Calculus of Variations, 30:17, 2024.

[2] F. Bagagiolo, R. Capuani and L. Marzufero. A zero-sum differential game for two opponent masses. Partial Differential Equations and Applications, 6 (2025), 19

[3] F. Bagagiolo and I. Romanò. State-constrained optimal control for a mass. In preparation.

Program

Regularizing effects of nonlinearity in balance laws

Laura Caravenna – Università di Padova

Although balance laws intrinsically describe phenomena where non-smooth solutions are expected, nonlinearity still has regularizing effects, as well known from the decay of positive waves in genuinely nonlinear families when there is conservation. This talk illustrates, through examples and key features, two such regularization phenomena occurring in balance laws, even in non-convex situations, when nonlinearity is 'non-degenerate'. This is particularly interesting because such fluxes are generic, in the sense of Baire category, among smooth fluxes.

In the context of solutions with small bounded variation for 1D systems, at all but at most countably many times, the solution is the sum of a step function and a $W^{1,1}$ function, ruling out the possibility of nasty fractal Cantor-like behaviors of the derivative.

For continuous solutions to a single balance law with bounded source, continuity improves to Hölder continuity, even in the multi-dimensional case.

Both results are based on Lagrangian approaches, albeit in very different senses. Based on recent joint works with Ancona, Cliffe, Marconi, Marson, Pinamonti, and precedent results involving Alberti, Bianchini, Bigolin, Serra Cassano.

Program

Two recent results based on hyperbolic conservation laws techniques

Rinaldo M. Colombo – Università di Brescia

First, we present an explicit formula expressing general entropy dissipation by means of Kruzkov entropy dissipation that applies to weak solutions to scalar balance laws. This result requires a careful choice of the functional setting.

Second, we show how systems of non local balance laws can be used in the encryption and decryption of various types of messages, such as texts, sounds or images. This procedure rises new analytical and numerical questions.

These results were obtained in collaboration with D. Amadori, M. Garavello, V. Perrolaz and A. Salvadori.

Program

The node transmission conditions approach to existence and stability for scalar conservation laws on a star shaped network

Carlotta Donadello – Université de Franche - Comté

We consider a wide family of Node Transmission Conditions (NTC), partially inspired by the membrane transmission conditions due to [Kedem, Katchalsky, Biochym. Biophys. Acta, 1958], on a star shaped network consisting of m incoming and n outgoing branches. Our attention focuses on NTC defined as operators (eventually multi-valued) relating the $(m+n)$-tuple of solution traces at the node to the $(m+n)$-tuple of the normal fluxes, which are $(m-T)$-accretive on $\mathbb{R}^{m+n}$ with respect to the norm $|\cdot|_1$. In this framework we show that the corresponding node germ, defined implicitly by a suitable projection procedure, is complete, maximal and L^1-dissipative. We prove existence of solutions to Cauchy problems via a convergent semi-discrete finite volume scheme, while their stability and uniqueness directly comes from the properties of the germ. In the specific setting of the work [Coclite, Donadello, NHM 2020] the solutions we obtain are characterized as vanishing viscosity limits – for the viscosity approximation obeying itself the underlying NTC.

Program

Spectral optimization for nonhomogeneous beams and fish-bone plates: some recent results

Maurizio Garrione – Politecnico di Milano

The maximization of the eigenvalues of nonhomogeneous multi-span structures of beam and fish-bone plate type, with respect to the mass density, is discussed. Some characterizations of the optimal densities are provided both for vertical and torsional eigenvalues, together with some hints regarding the relationships with the stability of suspension bridges.

Joint works with Elvise Berchio (Politecnico di Torino), Alessio Falocchi (Politecnico di Milano) and Clara Patriarca (Université Libre de Bruxelles).

Program

Unique solutions to hyperbolic conservation laws with a strictly convex entropy

Graziano Guerra – Università di Milano - Bicocca

Consider a strictly hyperbolic $n \times n$ system of conservation laws, where each characteristic field is either genuinely nonlinear or linearly degenerate. In this standard setting, it is well known that there exists a Lipschitz semigroup of weak solutions, defined on a domain of functions with small total variation. If the system admits a strictly convex entropy, we give a short proof that every entropy weak solution taking values within the domain of the semigroup coincides with a semigroup trajectory. The result shows that the assumptions of "Tame Variation" or "Tame Oscillation", previously used to achieve uniqueness, can be removed in the presence of a strictly convex entropy. Combined with a compactness argument, the result yields the existence of a uniform convergence rate for a very wide class of approximation algorithms. Some partial estimates on the convergence rate are given.

Program

The Camassa–Holm equation with transport noise

Helge Holden – Norwegian University of Science and Technology

We will discuss recent work regarding the Camassa–Holm equation with transport noise. In particular, we will show existence of a weak, global, dissipative solution of the Cauchy initial-value problem on the torus.

This is joint work with L. Galimberti (King's College), K.H. Karlsen (Oslo), and P.H.C. Pang (NTNU/Oslo).

Program

Nonlocal conservation laws on networks and routing

Alexander Keimer – Universität Rostock

In this talk, we will present some recent results on nonlocal conservation laws when applied to traffic flow networks with junctions.

We will first tackle well-posedness of the underlying model assuming that the junctions are modeled via buffers of arbitrary size. We then turn our attention to routing.

Instantaneous time routing will be considered as well as routing where traffic participants use the routes which give the minimal experienced travel time assuming that all other traffic participants aim for the same goal.

This can be formulated as a fixed-point problem in the proper Banach space and is an extension of well-known Wardrop's equilibrium (the Nash-equilibrium in traffic flow routing).

Some final remarks about optimal routing when all participants cooperate will conclude the talk.

Program

Wavefronts for a degenerate reaction-diffusion system with application to bacterial growth models

Luisa Malaguti – Università di Modena - Reggio Emilia

We consider a nonlinear system of two coupled reaction-diffusion equations with degenerate diffusivity introduced by Kawasaki et al. [1] for describing the spatial-temporal dynamics of a bacterial colony and its nutrient on an agar plate. In [1] the authors provided numerical evidence for the existence of wavefront solutions to the system leaving the analytical confirmation of these solutions an open problem. Muñoz-Hernández et al. [3] investigated a simplified version of the system proposed by Satnoianu et al [4] which corresponds to assume that the nutrient does not diffuse and did a complete discussion about the existence of wavefront solutions to their model. In [2] we return to the general, original model proposed in [1] and prove the existence of an infinite family of wavefronts parameterized by their wave speed, which varies on a closed positive half-line, thus confirming the conjecture in [1]. We provide an upper bound for the threshold speed and a lower bound for it when the diffusion coefficient is sufficiently large. The proofs are based on several analytical tools, including the upper and lower solutions approach, the shooting method and the fixed-point theory in Fréchet spaces, to establish existence, and the central manifold theorem to ascertain uniqueness.

References

[1] Kawasaki, K. and Matsushita, M. and Umeda, T. and Shigesada, N.; Modeling spatio-temporal patterns generated by Bacillus subtilis, J. Theor. Biol., 188 (1997).

[2] Malaguti, L and E. Sovrano; Wavefronts for a degenerate reaction-diffusion system with application to bacterial growth models, Journal of Differential Equations, 444 (2025).

[3] Muñoz-Hernández, E. and Sovrano, E. and Taddei, V.; Coupled reaction-diffusion equations with degenerate diffusivity: wavefront analysis, Nonlinearity, 38 (2025).
[4] Satnoianu, R. A. and Maini, P. K. and Garduno, F. S. and Armitage, J. P.; Travelling waves in a nonlinear degenerate diffusion model for bacterial pattern formation, Discrete Contin. Dyn. Syst. Ser. B, 1 (2001).

Optimally Controlled Moving Sets with Geographical Constraints

Elsa M. Marchini – Politecnico di Milano

The talk is concerned with a family of geometric evolution problems, modeling the spatial control of an invasive population within a plane region bounded by geographical barriers. The "contaminated region" is a set moving in the plane, which we would like to shrink as much as possible. To control the evolution of this set, we assign the velocity in the inward normal direction at every boundary point. Three main problems are studied: existence of an admissible strategy which eradicates the contamination in finite time, optimal strategies that achieve eradication in minimum time, strategies that minimize the average area of the contaminated set on a given time interval. For these optimization problems, a sufficient condition for optimality is proved, together with several necessary conditions. Based on these conditions, optimal set-valued motions are explicitly constructed in a number of cases.

Program

Vector-borne disease outbreak control via instant vector releases

Carlota Rebelo – Universidade de Lisboa

Vector-borne diseases have a large impact on human health around the world, representing 17% of all infectious diseases. These diseases can be due to parasites, bacteria or viruses and be transmitted by different types of vectors like, for instance, ticks, fleas or mosquitoes. This seminar is devoted to the study of optimal release strategies to control vector-borne diseases, such as dengue, Zika, chikungunya and malaria. Two techniques are considered: the sterile insect one (SIT), which consists in releasing sterilized males among wild vectors in order to perturb their reproduction, and the Wolbachia one (presently used mainly for mosquitoes), which consists in releasing vectors, that are infected with a bacterium limiting their vector capacity, in order to replace the wild population by one with reduced vector capacity. We will begin by describing the model without control strategies and then describe the results obtained for the two techniques.

Reference

[1] L. Almeida, J. Bellver Arnau, Y. Privat, C. Rebelo: Vector-borne disease outbreak control via instant vector releases, J. Math. Biol. 89 (2024), no. 6, Paper No. 63, 41 pp

Program

Coherence and Chattering in Isothermal Pipe Flow at Junctions

Massimiliano D. Rosini – Università di Chieti - Pescara

We investigate the behavior of isothermal flow through a network consisting of two pipes connected at a junction influenced by flow-modifying devices such as valves. A central focus is the development of a general framework for modeling coupling conditions at the junction, with particular attention to the notion of coherence. Coherence plays a critical role in determining the stability of the system and is directly related to the phenomenon of chattering – the rapid, repeated switching of devices like valves – which can destabilize numerical schemes used in simulation. We examine the coherence properties of several junction models, especially those representing control valves, and present numerical simulations that illustrate the emergence of chattering. Finally, we introduce a theoretical approach to eliminate chattering by ensuring model coherence, contributing to more robust and stable numerical computations in pipe network simulations.

Program

On a mixed ODE-PDE system: well posedness, stability and a differential game

Elena Rossi – Università di Modena - Reggio Emilia

In this talk we explore a zero-sum differential game governed by a mixed system consisting of an ODE and a hyperbolic balance law (PDE). This mixed system is inspired by epidemiology and models the conflict between two opposing actors: the scientific community and anti-vaccination groups. Each group seeks to influence public vaccination choices through their respective control strategies. We investigate the well posedness of the system for fixed control functions, the existence of optimal controls of each decoupled optimization problem, and the resulting differential game, based on a dynamic information pattern.

This talk is based on a joint work with Mauro Garavello (Università di Milano-Bicocca) and Abraham Sylla (Université de Picardie Jules Verne).

Program

BV curves of measures

Riccarda Rossi – Università di Brescia

Representation results for absolutely continuous curves with values in the Wasserstein space of Borel probability measures in $\mathbb{R}^d$ with finite p-moment, $p > 1$, provide a crucial tool to study evolutionary PDEs in a measure-theoretic setting. They are strictly related to the superposition principle for measure-valued solutions to the continuity equation.

This talk revolves around the extension of these results to the case $p = 1$, *and* to curves that are only of *bounded variation* in time.

Based on a joint collaboration with Stefano Almi (Napoli) and Giuseppe Savaré (Milano).

Program